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Regional modelling of ablation in West Greenland

by

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Abstract

Regional assessment of water resources in West Greenland is needed in connection with planning future hydropower projects. Ablation from the Inland Ice and from local glaciers will be an important water source in most basins and a simple model for computing specific ablation has been developed. The model uses air temperatures extrapolated from coastal weather stations and will be combined with areal data from the West Greenland glacier inventory to give estimates of runoff due to ablation for individual basins. The model is found to somewhat underestimate ablation at lower altitudes whilst greatly overestimating at higher altitudes and work will continue to improve the model. However, the present model applied to the Nordbogletscher in Johan Dahl Land indicates that the standard deviation of the ablation is about a quarter of the long-term mean which suggests that runoff measurements would have to be made for many years at the Nordbosø to obtain a reliable mean value for the runoff from the glacier.

CONTENTS

INTRODUCTION

There is currently a great interest in the exploitation of water resources in Greenland for the generation of hydroelectric power. This raises a number of problems. For example, information about hydrological conditions are very sparse compared to other areas where hydroelectric power is already being produced. Furthermore, a large part of the potentially available water will come from glacier ablation so that considerable glaciological insight and knowledge will be required.

Work is now in progress to improve the level of available information. The Greenland Technical Organization (GTO) has identified a number of basins as potential sites for hydropower projects and is now making hydrological and climatological measurements in them whilst the Geological Survey of Greenland (GGU) is attempting the assessment of available water resources on a regional basis.

This report describes a highly simplified but practical model for the computation of specific ablation which can be combined with areal data in the glacier inventory for West Greenland, also being made at the Geological Survey of Greenland, to compute basin runoff due to ablation. The present model is necessarily preliminary and neither the GGU nor the author of this report can take responsibility for any use of the current model for planning purposes. However, work will continue to improve the model and develop further regional models for other hydrological elements like precipitation and evaporation.

GLACIOLOGICAL AND HYDROLOGICAL CONSIDERATIONS

The specific balance b is the difference between the annual amount of ice and snow gained by the glacier surface (the specific accumulation c) and the amount lost (the specific ablation a):

$$b = c - a \tag{1}$$

Quantities in Equation (1) refer to a point on the glacier surface and are in kg m^{-2} a^{-1} or in mm a^{-1} for the equivalent depth of water. Ablation is usually defined to include all forms of mass loss, e.g. by melting, evaporation, calving of glacier tongues and wind erosion (Armstrong *et al.* 1973). However, under Greenland conditions, evaporation from glacier surfaces will be small compared to melting whilst calving and wind erosion will depend upon local conditions. For the present paper, the specific ablation will be treated as being practically synonymous with melting although some meltwater from snow surfaces may be refrozen within the surface layer of the glacier, e.g. superimposed ice formation. Ablation from the glacier surface will contribute to runoff from the glacier whilst rainfall on the glacier surface will also contribute. Much of the runoff will be routed within the glacier or along its bed and may be retained in temporary storage underway. This will have the effect of causing lags between peaks in ablation and rainfall and the resulting runoff peaks.

The quantities in Equation (1) will depend upon altitude and time so that, for example, a_{it} will be the specific ablation at altitude h_i in the year t. The specific quantities can be converted to total volumes of water by multiplication with the appropriate area. For example, if F_i is the glacier area lying below altitude h_i, the total ablation A_t will be given by:

$$A_t = \frac{1}{2} \sum_{i=1}^{i=m} (a_{it} + a_{i-1\,t})\,(F_i - F_{i-1}) \tag{2}$$

where h_O and h_m are the minimum and maximum altitudes of the glacier. The mean specific ablation $\bar{a}_t$ will be given by:

$$\bar{a}_t = A_t / F_m \tag{3}$$

where F_m is the total area of the glacier. Similar equations can be written for accumulation and balance. Neglecting evaporation from the glacier surface and changes in liquid storage, the total runoff from the glacier QG_t is given by:

$$QG_t = LP_t + A_t \tag{4}$$

where LP_t is the liquid precipitation on the ablation area. The total annual runoff Q_t from the whole basin is given by:

$$Q_t = PB_t - E_t - B_t \tag{5}$$

where PB_t is the total basin precipitation (liquid and solid), E_t is the basin evaporation and B_t is the volumetric balance of the glacier.

If the total runoff Q_t is fed into a high pressure tunnel at the altitude h_0 and allowed to fall more or less freely to a waterwheel at an altitude h the power generated will be:

$$W_t = \varepsilon \varrho g\, Q_t\, (h_0 - h) \tag{6}$$

where ε is an efficiency parameter, ϱ is the density of water (10^3 kg m^{-3}), g is the acceleration due to gravity (9.8 m s^{-2}) and $(h_0 - h)$ is the hydraulic head. Equation (6) illustrates the fact that the first hydrological task in connection with any hydropower project is the estimation of the available runoff Q_t. The fact that ablation will be a large source of runoff for most Greenland basins shows that this hydrological task has a considerable glaciological content.

Fluctuations from year-to-year in the runoff raise two problems. The first is that relatively long records of runoff are required to estimate reliable mean values and the second is that reservoirs must be overdesigned to store enough water in chance sequences of 'wet' years to maintain the desired outflow in any possible chance sequence of 'dry' years. The mean of an N–year runoff sequence Q_t ($t = 1$ *to* N) will be given by:

$$Q_{mn} = \frac{1}{N} \sum_{i=1}^{t=N} Q_t \tag{7}$$

whilst the standard deviation of the sequence will be denoted by S given by:

$$S^2 = \frac{1}{N-1} \sum_{t=1}^{t=N} [Q_t - Q_{mn}]^2 \tag{8}$$

The mean value calculated by Equation (7) will only be an estimate of the true mean of the population of runoff values from which the sequence Q_t is sampled. The true mean will have a 95 per cent probability of lying somewhere within the 95 per cent confidence interval around the estimated mean. Assuming that the observed runoff is sampled from a stationary random process, the confidence interval (Kreyszig, 1970, p. 178) is given by:

$$k = Sf/\sqrt{N} \tag{9}$$

where f is the value of Student's t for $N-1$ degrees of freedom at the 95 per cent confidence level. Some values of $f/\sqrt{N}$ are given in Table 1 from which it can be seen that the reliability of the estimated mean only increases slowly with the length of series N. If the year-to-year variations in runoff are actually autocorrelated rather than random, the confidence interval will be even wider than that given by Equation (9).

If water is taken from the reservoir at a constant rate equal to Q_{mn} whilst being filled by a random sequence of annual runoff values with mean value Q_{mn} and standard deviation S the capacity of the reservoir $CA(L)$ after L years will be given by the cumulative sum of the deviations of the runoff from the mean:

$$CA(L) = \sum_{t=1}^{t=L} Q_t - Q_{mn} \tag{10}$$

According to Hurst (1956) this cumulative sum will have a range from its maximum to its minimum given approximately by:

$$RA(L) = S(L/2)^{0.73} \tag{11}$$

If the reservoir is filled to a capacity of RA before the start of operations one should be able

Table 1. The 95 per cent confidence interval for the mean of a normal distribution with sample standard deviation S

After Kreyszig (1970, p. 178)

Sample size	Confidence interval
5	±1.24 S
6	±1.05 S
7	±0.93 S
8	±0.84 S
9	±0.77 S
10	±0.72 S
20	±0.47 S
100	±0.20 S
500	±0.09 S

to withdraw an amount of water equal to Q_{mn} for L years without the reservoir running dry because sequences of deficits will be compensated by sequences of surpluses. Mathematical aspects of reservoir theory are still being debated (Lloyd, 1974) but knowledge of the statistical properties of the runoff series will always be needed.

The foregoing is based upon the assumption that the runoff is stationary, i.e. although the runoff fluctuates from year to year its statistical properties remain constant. This need not be true over a long period. For example, the runoff may change over the years because of a change in specific ablation due to climatic change (induced variation) or due to secular changes in the glacier area due to past changes in specific ablation (relaxation variation). Secular variations in basin precipitation may also occur.

The question of predicting induced variations in runoff is equivalent to predicting future climatic variations which may never be possible. With regard to the relaxation variations, it is true that a glacier will become smaller in the future if its present balance is negative but there will be a delay and the problem is a difficult one from the mathematical point of view (Paterson, 1969). An important corollary to the difficulty of predicting runoff changes due to naturally occurring glacier changes is that those due to artificial changes will be equally difficult to predict so that glacier 'modification' should not be undertaken lightly.

STRATEGY FOR A REGIONAL MODEL

Information about ablation in West Greenland is very scattered; see Drygalski (1897), de Quervain & Mercanton (1925), Loewe & Wegener (1933), LaChapelle (1955), Kuhlman (1959) and Ambach (1963). Few of the data refer to a whole year. There are only very few short runoff series from glacier-covered basins in West Greenland, e.g. Larsen (1973). Runoff and ablation data will be required for the rational planning of any future hydroelectric project whereby runoff measurements should extend over as many years as possible in any basin which is being considered seriously for exploitation. However, such measurements are expensive to make and there is danger that unsuitable basins will be chosen because of the lack of prior information. However, this will only become apparent after the costly measurements have been made.

The best solution to the problem is to first develop general regional models which will give approximate information about all basins from which a selection of basins for further study can be made. As more reliable data for these test basins become available, the models can be tested and improved until they agree with the improved data base for the test basins. At this stage the model should also give useful information about basins which have not been studied and will serve as a rational basis for selecting basins for the development of hydroelectric power. The flowsheet in fig. 1 describes the development of such a model schematically. The problem of assessing basin runoff is decomposed into the seperate problems of estimating specific ablation and precipitation on the one hand and of measuring basin characterstics on the other hand. This division merely reflects the different controls of climate and topography on the runoff.

An important step in implementing the model has already been taken by Weidick & Olesen (1978) who made an inventory of hydrological basins in West Greenland and estimated their water balance. Figures for the latter are summarized in Table 2 from which it

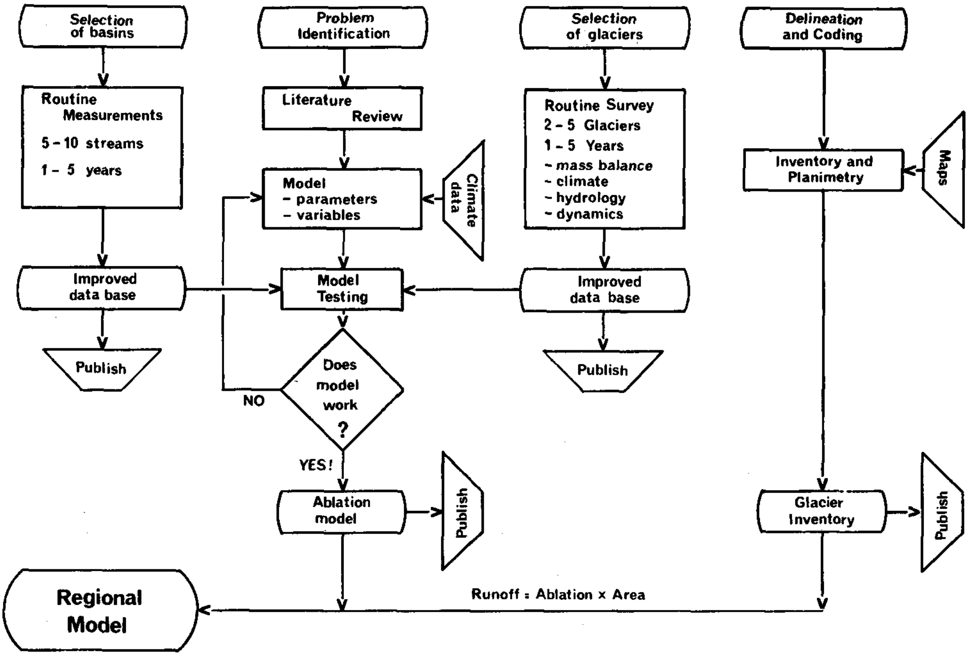

Fig. 1. Flowsheet describing development of a regional ablation model by the Geological Survey of Greenland (GGU).

Table 2. Water balance of West Greenland (484 000 km²)

Sources and Sinks	Amount	
Precipitation		
Inland Ice accumulation area (337 000 km²)	+157	
Inland Ice ablation area (61 000 km²)	+27	
local glaciers in coastal basins (13 000 km²)	+6	
ice-free coastal areas (73 000 km²)	+27	
Total water sources	+217	$km^3\,a^{-1}$
Ablation		
Inland Ice ablation area (61 000 km²)	-60	
local glaciers in coastal basins (13 000 km²)	-6	
Direct runoff from summer snowmelt and rainfall		
Inland Ice ablation area (61 000 km²)	-27	
ice-free coastal areas (73 000 km²)	-27	
Calf ice production	-97	
Total water sinks	-217	$km^3\,a^{-1}$

According to Weidick & Olesen, 1978.

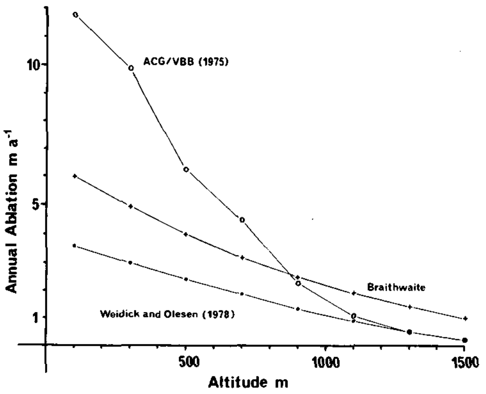

Fig. 2. Comparison between three ablation models: the ACG/VBB (1975) model for latitude 60–63° N, the Weidick & Olesen (1978) model and the present model (Braithwaite) using data from Narssarssuaq.

can be seen that calf ice production is the major water sink. However, not all the remaining 120 km^3 a^{-1} of water will be potentially available for exploitation as 23 km^3 a^{-1} of ablation and 7 km^3 a^{-1} of direct runoff from summer snowmelt and rainfall originate in calving sectors of the Inland Ice, i.e. they are available with a hydraulic head of zero. This then reduces the theoretically usable amount of water in West Greenland to only 41 per cent of the total precipitation income. The estimates of Weidick & Olesen (1978) are still rather crude and some terms have been neglected, e.g. evaporation from ice-free areas, but they do prompt the thought that earlier estimates of the 'power potential' of the Greenland Inland Ice have been exaggerated by overestimation of available water.

Work is now in progress at the Geological Survey of Greenland to refine the basin inventory of Weidick & Olesen (1978) by the compilation of a glacier inventory to internationally agreed standards as well as improvement in the estimates of water balance for individual basins. At the same time, detailed mass balance and local climate studies have been started on the Nordbogletscher in Johan Dahl Land (latitude 61 °N, Inventory number: 1AG05001) in 1977 (Olesen & Weidick, 1978) and on the Qamanârssûp sermia near Godthåb (64 °N, 1CH21002) in 1979 (Olesen, personal communication) to improve the available data base. The task of the author of the present report is the improvement of the estimates of specific ablation used in the scheme in fig. 1. The need for such improvement can be illustrated by comparison of three current models in fig. 2. They are the one assumed in ACG/VBB (1975) for the latitude zone 60 to 63 °N, the model of Weidick & Olesen (1978) based upon measurements by Loewe & Wegener (1933) and the model described in the following section using temperature data from Narssarssuaq. The wide differences between the models are clear.

THE ABLATION MODEL

The proposed ablation model is based upon the one developed for the White Glacier, Axel Heiberg Island by Braithwaite & Müller (1976) and Braithwaite (1977) although the parameter values will have to be adjusted in the future in the light of experience in West

Greenland. The model assumes that the ablation is proportional to the mean of the positive temperatures during the same period and at the same place (Finsterwalder & Schunk, 1887):

$$a = N^+\alpha_1 + \beta_1 N T^+_{mn} \tag{12}$$

where a is the specific ablation during a period of N days when the mean of positive temperatures in the same period is T^+_{mn}. Positive temperatures occur on a total of N^+ days during the period. The empirical parameters α, and β, are to be determined from field observations. The factor $N\,T^+_{mn}$ is simply the degree day factor or positive temperature sum of Zingg (1952), Kasser (1959) and Orheim (1970). A justification of Equation (12) from energy balance considerations is given by Braithwaite (1977). The factors N^+ and T^+_{mn} can be readily evaluated from time series of daily mean temperatures, i.e. N^+ is obtained by counting the number of days when temperature is 0 °C or greater whilst T^+_{mn} is calculated from:

$$T^+_{mn} = (1/N) \sum_{t=1}^{t=N} {}^{+} T_t \tag{13}$$

where the '+' sign on the summation denotes that only positive values of daily mean temperature T_t are summed. In practice the computation of T^+_{mn} according to Equation (13) is very laborious if data for many years are used so it is estimated from the following:

$$T^+_{mn} = \int_0^{T_{max}} T f(T)\, dT \tag{14}$$

whilst N^+ is evaluated from:

$$N^+ = \int_0^{T_{max}} f(T)\, dT \tag{15}$$

where $f(T)$ is the probability density function describing the statistical distribution of daily mean temperatures during the period of length N days. Strictly speaking, the replacement of the time summation in Equation (13) by the ensemble integration in Equation (14) is only valid for a stationary process so the period of N days is chosen to be a calender month during which it is fair to assume that the synoptic variations of temperature still predominate over the seasonal trend. For the present, the density function $f(T)$ is assumed to be that of the Gaussian distribution (Kreyszig, 1970) with a monthly mean temperature of T_{mn} and standard deviation of ± 4 °C whilst T_{max} is taken to be T_{mn} plus 8°C, i.e. two standard deviations above the mean. The mean positive temperature T^+_{mn} only becomes identical whith the monthly mean temperature T_{mn} for very warm months and is zero or small for very cold months.

Combining Equations (14) and (15) with Equation (12) allows the calculation of the monthly ablation at any point from the corresponding monthly mean temperature. The latter must be extrapolated from a permanent weather station located on the coast of

Greenland at approximately the same latitude as the point in question. This is done according to the equation:

$$T_{mn} = \alpha_2 + \beta_2 \left[T_s + C + g(h-h_s) \right] \tag{16}$$

where T_s is the monthly mean temperature at a coastal station at altitude h_s whilst h is the altitude of the point in question, g is the vertical lapse rate of temperature, C is a factor representing the heating of the air as it moves inland (continentality effect) whilst α_2 and β_2 are empirical parameters describing the cooling of the air as it moves over the glacier.

In principle, the proposed model can be used to compute ablation for any point for any month on the basis of the monthly mean temperature T_s at a coastal weather station if the parameters α_1, β_1, α_2, β_2, C and g are known. Their values can be obtained from field observations if such are available whilst, at the same time, the hypotheses expressed by Equations (12) and (16) can also be tested. The parameters α_1 and β_1 can be computed by comparing series of daily ablation rate with temperature series at the same place whilst α_2, β_2, C and g can be calculated from daily temperature data measured in a local network of thermograph stations sampling both glacier-covered and glacier-free locations at various altitudes, e.g. a network like the one described by Ohmura & Müller (1977). As a file of parameter values for different conditions is built up it may be possible to further refine Equation (16) to take account of variations between localities or between different sequences of weather.

Results using the model for the White Glacier, Axel Heiberg Island, can be briefly quoted, further details and preliminary results are given by Braithwaite & Müller (1976) and Braithwaite (1977). The parameters assumed in the current model are:

$$\begin{aligned} \alpha_1 &= 0 \text{ mm d}^{-1} \\ \beta_1 &= 6 \text{ mm d}^{-1} \text{ deg}^{-1} \\ \alpha_2 &= -0.7 \text{ deg} \\ \beta_2 &= 0.83 \end{aligned}$$

The input data to the model were upper air data at two nearby weather stations (at distances of 113 and 280 km) whilst the lapse rate g was evaluated directly from the upper air data.

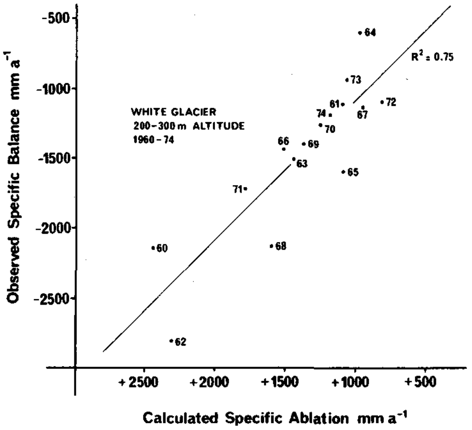

Fig. 3. Comparison between observed specific balance and calculated specific ablation at 200–300 m on the White Glacier, Arctic Canada. Observed data from Müller (1977).

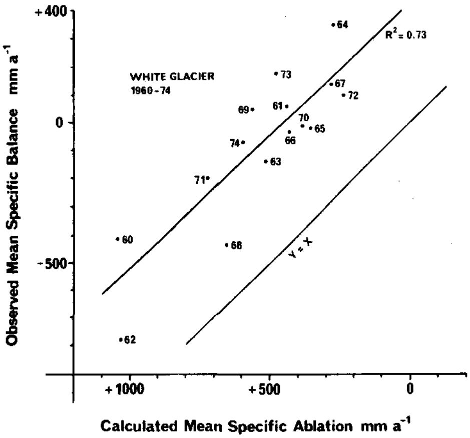

Fig. 4. Comparison between observed mean specific balance and calculated mean specific ablation on the White Glacier, Arctic Canada. Observed data from Müller (1977).

The continentality C was found to be about zero for the White Glacier. Annual ablation was computed for every year in the period 1959/60 to 1973/74 at 100 m altitude intervals. Results for the computed specific ablation for the altitude band 200–300 m are given in fig. 3 in comparison with the latest values for the observed specific balance given by Müller (1977); seperate accumulation and ablation measurements have not yet been published for the White Glacier. From fig. 3 it can be seen that the correlation between computed ablation and observed balance is quite good ($R^2 = 0.75$) corresponding to an error standard deviation of ± 280 mm a^{-1}. The mean difference between the two quantities is only about 80 mm a^{-1} which is five per cent of the 15-year mean of the observed balance. One reason for this good agreement is that the accumulation at the altitude 200–300 m is very small. Computed mean specific ablation (referring to the whole glacier) is compared to the observed mean specific balance in fig. 4 from where it can be seen that the correlation is almost as good ($R^2 = 0.73$) as in the previous case but there is a bigger average difference of 430 mm a^{-1}. This seems to be too large to be solely due to the mean specific accumulation (for which precise values are not available) and suggests that the model probably overestimates the ablation in the upper parts of the glacier. This could be because superimposed ice has been neglected in the model which also assumes that snowmelt and icemelt are equally efficient. However, it is also possible that the model underestimates the magnitude of the cooling effect in the upper parts of the glacier.

PRELIMINARY RESULTS FOR WEST GREENLAND

Suitable data to implement the proposed model *sensu stricto* for West Greenland do not exist yet. However, it was judged useful to run the model with the same parameters as the White Glacier case, described in the previous section.

The model described by Equations (12) to (16) was used to calculate annual specific ablation at 200 m altitude intervals between 100 and 1500 m. The vertical lapse rate g was assumed to be -0.006 deg m^{-1} whilst the parameter C was neglected for the present. Input

Table 3. Weather stations on the west coast of Greenland operated 1965–1974 by the Danish Meteorological Institute (DMI)

Station	Lat.	Long.	Neighbour	Distance
Thule (Qânâq)	77°29′	69°12′	Dundas	100 km
Dundas	76°34′	68°48′	Thule	100 km
Upernavik	72°47′	56°10′	Umanak	280 km
Umanak	70°40′	52°00′	Qutdligssat	80 km
Qutdligssat	70°03′	52°51′	Umanak	80 km
Godhavn	69°14′	53°31′	Egedesminde	70 km
Jakobshavn	69°13′	51°03′	Christianshåb	40 km
Christianshåb	68°49′	51°05′	Jakobshavn	40 km
Egedesminde	68°42′	52°45′	Christianshåb	70 km
Holsteinsborg	66°55′	53°40′	Sukkertoppen	170 km
Sukkertoppen	65°24′	52°52′	Godthåb	150 km
Godthåb	64°10′	51°45′	Færingehavn	50 km
Færingehavn	63°42′	51°33	Godthåb	50 km
Frederikshåb	62°00′	49°43′	Færingehavn	210 km
Narssarssuaq	61°11′	45°25′	Narssaq★	50 km
Narssaq★	60°54′	45°58′	Julianehåb	20 km
Julianehåb	60°43′	46°03′	Narssaq★	20 km
Nanortalik	60°08′	45°13′	Julianehåb	80 km

★ operation discontinued by DMI in 1969/1970.

Table 4. Ten-year mean ablation in mm a^{-1} computed from temperature data at weather stations on the west coast of Greenland for the period 1965–1974

	Altitude (m)							
Station	100	300	500	700	900	1100	1300	1500
Thule (Qânâq)	1610	1240	880	620	420	280	170	100
Dundas	1540	1150	820	570	380	240	150	90
Upernavik	2150	1620	1200	860	600	400	260	160
Umanak	3520	2820	2210	1690	1250	900	620	420
Qutdligssat†	2850	2210	1680	1250	890	610	410	260
Godhavn	3010	2360	1790	1330	960	670	450	290
Jakobshavn	3440	2740	2150	1630	1210	860	590	390
Christianshåb	3940	3200	2540	1980	1500	1100	780	540
Egedesminde	2600	1990	1480	1070	750	500	330	200
Holsteinsborg	2950	2280	1720	1250	890	610	400	250
Sukkertoppen	3590	2810	2150	1610	1170	820	560	360
Godthåb	3400	2620	1980	1450	1030	710	470	300
Færingehavn†	2970	2270	1680	1210	850	570	370	230
Frederikshåb	3260	2450	1800	1280	880	590	370	230
Narssarssuaq	6030	4970	4030	3200	2480	1880	1390	990
Narssaq★	5210	4170	3290	2540	1910	1400	990	690
Julianehåb	4690	3680	2820	2110	1530	1080	730	480
Nanortalik	4660	3590	2700	1970	1380	940	620	390

★ data from DMI and GGU stations combined.
† full 10-year record not available.

data to the model were monthly mean temperatures in the ten-year period 1965–74 at 18 permanent weather stations along the west coast of Greenland. The stations were operated by the Danish Meteorological Institute (DMI) and the data have been published in the DMI's annual reports entitled 'Provisional temperatures and total amount of precipitation in mm, Greenland'. The names and locations of the stations are given in Table 3 together with the distance of each station from its nearest neighbour. The overall impression is that the station density is moderately high although the coverage is obviously biased to coastal conditions.

Ten-year averages of the computed ablation in mm a^{-1} and the length of the ablation season in days at different altitudes are given in Tables 4 and 5. The results must be treated with caution because the effects of continentality represented by the parameter C have been neglected in the model whilst the temperatures at the various stations will be affected to greater or lesser degree by the influence of the sea. This means that the ablation and the length of ablation season are both somewhat underestimated at lower altitudes at most stations. On the other hand, they are both probably overestimated at higher altitudes as in the White Glacier case.

Despite scatter and individual 'anomalies' the computed values in Tables 4 and 5 show clear trends of decreasing ablation and shorter ablation seasons with increasing latitude. The linear trends are plotted in figures 5 and 6 but it should be noted that the correlations with the trends become weaker at higher altitudes. From fig. 5 it can be seen that the ablation at 100 m decreases from roughly 5700 mm a^{-1} at 60°N to about 2900 at 70°N and only about

Table 5. Ten-year mean length of ablation season in days computed from temperature data at weather stations on the west coast of Greenland for the period 1965–1974

	Altitude (m)							
Station	100	300	500	700	900	1100	1300	1500
Thule (Qânâq)	71	59	47	37	27	20	13	9
Dundas	71	58	46	36	26	18	12	8
Upernavik	92	76	62	49	37	27	19	13
Umanak	122	107	92	78	63	50	38	28
Qutdligssat[†]	109	94	78	64	51	39	28	20
Godhavn	115	99	82	67	54	41	30	21
Jakobshavn	120	106	90	76	62	49	37	27
Christianshåb	128	114	99	85	71	57	45	34
Egedesminde	108	91	74	59	45	33	24	16
Holsteinsborg	117	100	83	67	52	39	28	20
Sukkertoppen	136	116	97	79	63	49	36	26
Godthåb	137	115	95	76	59	45	32	23
Færingehavn[†]	124	104	84	67	51	38	27	18
Frederikshåb	142	117	94	74	55	40	28	19
Narssarssuaq	182	163	144	125	107	89	72	57
Narssaq[★]	177	155	132	112	92	73	57	43
Julianehåb	176	151	127	104	83	64	47	34
Nanortalik	187	159	132	106	82	61	44	30

[★] data from DMI and GGU stations combined.

[†] full 10-year record not available.

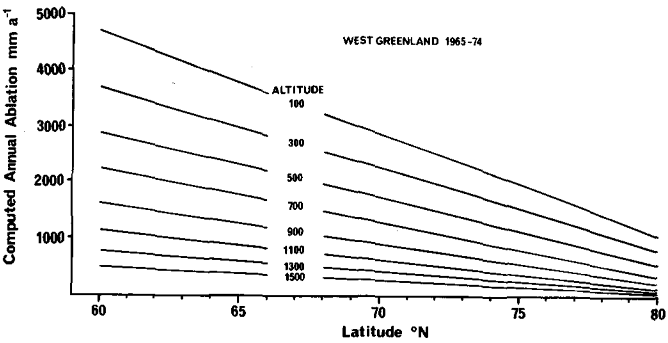

Fig. 5. Computed annual ablation for 1965–1974 in West Greenland in mm a^{-1}.

1100 mm a^{-1} at 80°N. The ablation gradient, defined simply as the slope of the regression equation for computed ablation versus altitude in the present case, is also highest in the south with a value of about –3 mm a^{-1} m^{-1} at 60°N, dropping to only about –1 mm a^{-1} m^{-1} at 80°N. However, it must be remembered that these figures will be too low if the ablation is really underestimated by the model at lower altitudes and overestimated at higher altitudes whilst the ablation versus altitude relation itself becomes more inflected at lower temperatures, i.e. with increasing latitude. From fig. 6 it can be seen that the computed length of the ablation season at 100 m drops from about 170 d a^{-1} at 60°N to about 110 d a^{-1} at 70°N and only about 50 d a^{-1} at 80°N.

The results of the model can be examined in more detail by considering the case of the Nordbogletscher (glacier 1AG05001) in Johan Dahl Land (basin JHB,G,5.0) in southern Greenland. The glacier is an outlet of the Inland Ice with an estimated area of 208 km² lying between 600 and 2300 m (Olesen & Weidick, 1978) which may be expected to provide runoff by ablation to the Nordbosø where a hydropower project is planned (ACG/VBB, 1975). Runoff measurements have been made in the basin for several years by the Greenland Technical Organization (GTO) whilst the Geological Survey of Greenland (GGU) has been making local climate and mass balance measurements on the glacier tongue between 780 and 920 m since 1977 (Olesen & Weidick, 1978).

The nearest permanent weather station to the Nordbogletscher is Narssarssuaq a few kilometres to the south. As this station lies well inland and is much warmer than neighbour-

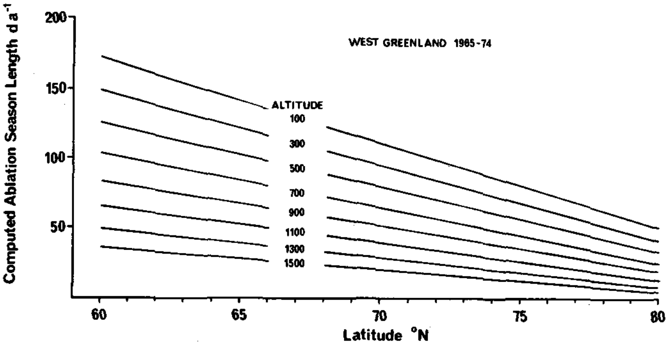

Fig. 6. Computed length of ablation season for 1965–1974 in West Greenland in d a^{-1}.

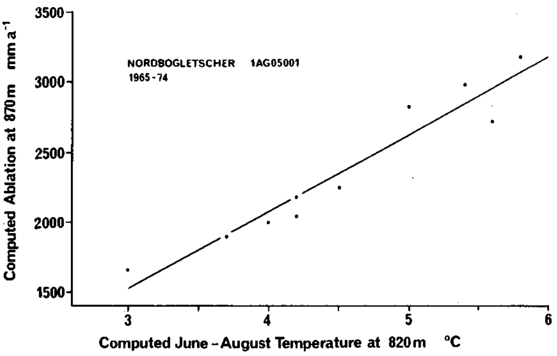

Fig. 7. Comparison between computed specific ablation at 870 m on the Nordbogletscher and computed mean summer (June to August) temperature at 820 m beside the glacier.

ing stations like Narssaq, Julianehåb and Nanortalik it might be expected that its temperature record already includes the effects of heating by the land. If this is the case, results for the model using Narssarssuaq data should be valid for the Nordbogletscher. From the values in Table 4, the ten-year average ablation for 1965–74 using Narssarssuaq data should be about 2590 mm a^{-1} at 870 m which is the mean altitude of the 27 stakes on the glacier. By contrast, the observed values were 2640 ± 250 mm a^{-1} for 1978 (Olesen & Weidick, 1978) and 2160 ± 340 mm a^{-1} for 1979 (Clement, 1980). Alternatively, the observed summer (June to August) mean temperatures for 1978 and 1979 at the base camp weather station at 820 m can be entered into the regression line in fig. 7 which shows the relation between computed ablation at 870 m and computed summer (June to August) mean temperature at 820 m according to the model. The temperatures are 4.3 and 3.7°C respectively which give computed ablation values of 2240 and 1910 mm a^{-1} for 1978 and 1979 respectively. These values are respectively 15 and 12 per cent lower than the corresponding observed values. These underestimations will mainly reflect the combined effects of the assumed $\alpha_1, \beta_1, \alpha_2$ and β_2 parameters in the model because errors in g, as well as due to the neglect of C, have been largely excluded by entering measured temperatures for the two years into the regression equation.

The total volumetric ablation for Nordbogletscher up to 1800 m was calculated for each year in 1965–74 by combining the specific ablation values for the Narssarsuaq data with the hypsographic data given by Olesen & Weidick (1978). The ten-year mean of the total

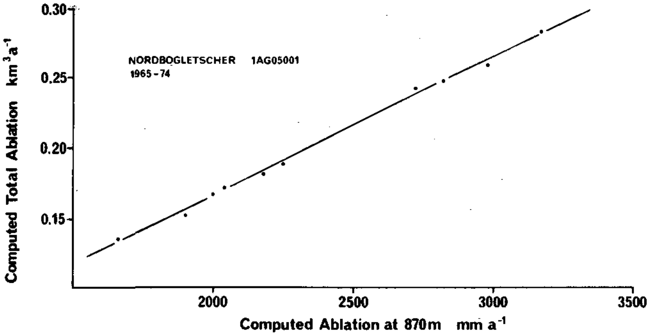

Fig. 8. Comparison between computed total ablation on the Nordbogletscher and computed specific ablation at 870 m.

ablation is 0.20 km^3 a^{-1} with a standard deviation of 0.05 km^3 a^{-1}. Fig. 8 shows that, according to the model, the total ablation should be closely related to the specific ablation at 870 m, i.e. that the specific ablation at any particular altitude should be an 'index' of the total ablation of the whole glacier. Entering the observed values of specific ablation at 870 m for 1978 and 1979 (values of 2640 and 2160 mm a^{-1} respectively) gives estimates of 0.23 and 0.18 km^3 a^{-1} for the total ablation in the two years. These values are actually higher than the observed runoff values at Nordbosø of 0.15 km^3 a^{-1} for 1978 (GTO, 1979) and 0.14 km^3 a^{-1} for 1979 (GTO, personal communication) which include precipitation as well as runoff due to ablation. This shows that the model still grossly overestimates the total ablation of the Nordbogletscher although, as already said, it somewhat underestimates the ablation at 870 m. The poor performance of the model with respect to total ablation almost certainly arises because the model overestimates ablation in the higher parts of the glacier although it is also possible that the delineation of the glacier is in error so that part of the glacier runoff does not in fact drain into the Nordbosø at all.

The above example shows that there is considerable room for improvement in the model for which purpose field data, both ablation measurements and runoff, will be most valuable. Although the computed total ablation, with a ten-year mean of 0.20 km^3 a^{-1} with a standard deviation of 0.05 km^3 a^{-1}, is certainly an overestimate it may still be true that the standard deviation constitutes about a quarter of the mean of the correct values. If this is the case, according to Equation (9) the runoff measurements at the Nordbosø would have to be made for more than 20 years to reduce the 95 per cent probable error in the mean to less than a tenth of the mean itself. At the same time, according to Equation (11) the maximum capacity of the reservoir would have to be about 1.3 to 1.4 times the true mean yearly runoff to ensure a 20-year operation. In order to take account of the fact that the mean runoff during this operation period may be greater or less than the previously measured runoff, it would be wise to figure on only using about 80 to 90 per cent of the measured mean runoff.

The above figures refer to a stationary climate. However, the model also predicts that the mean ablation will change with a change in mean summer temperature (induced response). Combining the regression lines in figures 7 and 8 gives a temperature response of about 0.05 km^3 a^{-1} deg^{-1} for the total ablation, i.e. the mean ablation will change by about a quarter of the value under present climate for a temperature change of 1°C.

In the original plan for the Nordbosø project (ACG/VBB, 1975) the available runoff was estimated to be 0.25 km^3 a^{-1} whilst the mean runoff for the four available years 1976–79 (GTO, 1979 and GTO, personal communication) is only 0.14 km^3 a^{-1}, i.e. 44 per cent less than the original estimate. These figures show the vital need to continue the measurements while it must be hoped that the ablation model can be improved to a degree that it can be used to construct a synthetic runoff series predating the start of the measurements. This would allow the final plan for the Nordbosø project and, most important, the estimation of its productivity for marketing purposes to be based upon the most complete information available.

FUTURE OUTLOOK

The present ablation model can be criticised on a number of grounds. First, the heating of the air as it moves inland is neglected. Second, the cooling of the air as it moves over the glaciers is described simply by two constant parameters whilst one might expect that the cooling should increase as the air moves further in from the ice edge, i.e. the cooling should be stronger in the upper parts of the glaciers than on the tongues. Third, an identity has been assumed between melting and ablation although a snowcover may be expected to retain meltwater *in situ* by ripening, ponding and formation of superimposed ice so that not all of the computed meltwater can actually contribute to runoff. Fourth, the melting of snow and ice surfaces are assumed to be identical under the same external conditions although the snow surface should receive less energy than an ice surface because of the higher albedo and lower surface roughness. Last, there is the fundamental point that the model describes a complex system, involving many interacting variables, in terms of only one variable the air temperature, with all other variables being 'lumped' into a few simple parameters.

Despite the above, the results already obtained with the model suggest that an adequate model can be achieved if at least some of the criticisms can be overcome. For example, in the 1980 fieldwork it is planned to extend the stake network on the Nordbogletscher into the upper parts of the glacier (Clement, personal communication) so that it should be possible to include the effects of varying snowcover in the model in future. Similar results from the new GGU project on the Qamanârssûp sermia will improve the regional aspects of the model.

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