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A simple model of runoff from ungauged basins in  
West Greenland

by

*Roger J. Braithwaite*

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## **Abstract**

A simple model of runoff in West Greenland is proposed for the case that glaciers are in a state of balance. The independent variables in the model are estimated annual precipitation and evaporation while model parameters are evaluated by analyses of precipitation series from Greenland, and mass balance series from other parts of the world. The model can be applied to ungauged basins for preliminary calculations of mean runoff, reservoir capacities and relative error in mean runoff.

Application of the model to sixteen proposed hydropower projects in West Greenland shows an order-of-magnitude agreement with earlier estimates. However, a more detailed analysis of runoff from Johan Dahl Land confirms that the runoff there is less than originally estimated. The proposed reservoir appears to be adequate for 30-year storage but the design yield of the project must be reduced for safety.

The shortcomings of the model should be obvious. They can be best overcome by an improved knowledge of hydrological conditions in the country resulting from systematic field observations.

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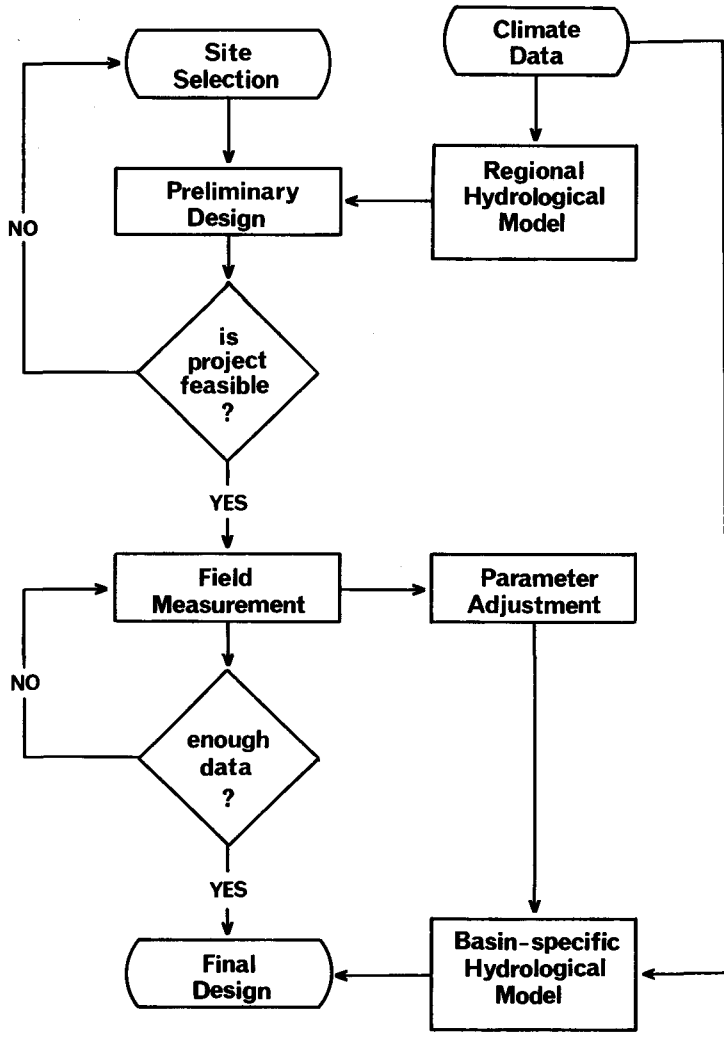


Fig. 1. Illustration of the relationship between hydrological models and the design of hydropower projects.

## BACKGROUND

The possibility of generating hydropower in Greenland has received growing attention in the last decade. The first serious proposal for establishing hydropower stations in Greenland was made by ACG/VBB (1975). The names and locations of the sixteen basins that were proposed are given in Table 1 together with the corresponding estimates of runoff and power production. However, the figures given are only rough estimates as there were few hydrological observations at the time. Indeed, the proposal by ACG/VBB (1975) was the necessary stimulation for the investigations which have been made since 1975.

The first attempt at a systematic regional assessment of water resources in Greenland, albeit only for the populous and accessible southwestern part, was made by Weidick & Olesen (1978, 1980) as part of a basin inventory. Areas and physiographic characteristics were determined for a total of 870 basins, covering an area of 484 000 km<sup>2</sup>, and rough estimates of water balances were made using simple regional models.

Field studies on the climate and mass balance of three glacier areas have been started; the Nordbogletscher in Johan Dahl Land (Olesen, 1978; Clement & Olesen, 1980; Clement 1981, 1982), Qamanârssûp sermia in Godthåbsfjord (Olesen, 1981; Olesen & Braithwaite, 1982) and Tasersiaq between Godthåb and Søndre Strømfjord (Olesen, 1982). In addition to these studies by the Geological Survey of Greenland (GGU), the Greenland Technical Organization (GTO) is making runoff and climate measurements in a number of basins (GTO, 1979) while private industry and Danish universities are becoming increasingly involved.

The work described in the present report is intended as an extension of the approach of Weidick & Olesen (1978, 1980). It concerns the development of a regional hydrological model which purports to calculate mean runoff from ungauged basins. It also attempts, for

*Table 1. Estimated runoff and power production of sixteen hydroelectric projects proposed by ACG/VBB (1975)*

Project	District	Latitude	Runoff km <sup>3</sup> a <sup>-1</sup>	Production GWh a <sup>-1</sup>
Motzfeldt Sø	Narssaq	61°10'	0.440	110
Johan Dahl Land	Narssaq	61°15'	0.250	370
Grænseland	Frederikshåb	61°20'	0.440	460
Isorrssua	Frederikshåb	61°40'	0.360	580
Kangarssup	Frederikshåb	62°30'	1.800	540
Bjørnesund	Godthåb	63°00'	0.270	390
Qaqat akulerit	Godthåb	63°15'	0.390	390
Grædefjord	Godthåb	63°25'	1.275	860
Isortuarssûp	Godthåb	63°40'	0.950	960
Buksefjorden	Godthåb	63°55'	0.220	120
Imarssuaq	Godthåb	64°50'	0.820	1230
Taserssuaq	Godthåb	64°55'	3.300	390
Søndre Isortoq	Sukkertoppen	65°40'	1.000	880
Tasersiaq	Sukkertoppen	66°25'	1.660	2100
Umivit	Sukkertoppen	66°50'	1.320	690
Nûgssuaq	Jakobshavn	70°05'	0.180	100

the first time, the estimation of the coefficient of variation of the runoff which is used for calculation of reservoir size and sampling error for the mean runoff.

The idea of trying to calculate runoff from basins where no measurements have been made may seem heretical. However, it is necessary in Greenland at present. This is because there are few runoff records while there is a great, and urgent, interest in exploiting the runoff. In practice, this means that the identification of possible sites for hydropower projects, and their preliminary design, must be made in the absence of direct data, i.e. by using a regional hydrological model as shown in fig. 1. Programmes of field measurement of runoff will then only be started in those basins which are identified as attractive possibilities for exploitation. After a few years of measurements have been made, the resulting data can be used for the final design of the project, possibly with the help of a basin-specific model which is calibrated with the measured data and then used to make simulations based upon long-term climatic data.

Naturally, regional hydrological models cannot, in the nature of things, be highly accurate. However, they only need to be accurate enough to avoid the danger of making incorrect decisions about which localities are worth the expense of further study.

## THE MODEL

The model refers to a basin of total area  $F_B$  which is partly covered by glacier ice. The distribution of annual precipitation within the basin is assumed to be uniform whilst melting is the only form of ablation considered, i.e. calving is excluded and evaporation from glacier surfaces is neglected in comparison to evaporation from ice-free areas. The specific annual runoff  $q_t$  can be written as:

$$q_t = p_t - \alpha b_t - (1-\alpha)e_t \quad (1)$$

where the subscript  $t$  denotes the  $t$ th year,  $p_t$  is the specific annual precipitation,  $b_t$  the specific annual balance of the glaciers in the basin,  $e_t$  the specific annual evaporation from ice-free areas and  $\alpha$  is the degree of glacierization of the basin. Equation (1) treats the variables as annual quantities so that seasonal variations are not described. The equation also assumes that delay times in the internal routings within the basin are short compared to a year so that there is no carry-over of water from one year to the next inside the basin.

The total runoff from the basin is given by:

$$Q_t = q_t F_B \quad (2)$$

where  $F_B$  is the basin area.

In principle the precipitation, glacier balance and evaporation will vary from year to year to produce a varying runoff. It is assumed that all these quantities are stationary random processes, i.e. that they fluctuate by chance around constant mean values. The mean runoff  $\bar{q}$  for a record of length  $N$  will be given by:



$$\bar{q} = (1/N) \sum_{t=1}^{t=N} q_t \quad (3)$$

while the corresponding standard deviation  $s_q$  will be given by:

$$s_q^2 = (1/N-1) \sum_{t=1}^{t=N} (q_t - \bar{q})^2 \quad (4)$$

The mean values  $\bar{p}$ ,  $\bar{b}$  and  $\bar{e}$  and the standard deviations  $s_p$ ,  $s_b$  and  $s$  are defined in similar ways for precipitation, glacier balance and evaporation respectively.

An important simplification is introduced by postulating that the glaciers in the basin are in a long-term state of balance, i.e.  $\bar{b} = 0$ , although the balances in the individual years will not be zero. The mean runoff with a mean balance of zero is denoted by  $\bar{q}_0$  which is given by:

$$\bar{q}_0 = \bar{p} - (1 - \alpha)\bar{e} \quad (5)$$

There is in fact no direct evidence that the assumption of zero balance is correct but it is at least a 'neutral' assumption and a useful one from the point of view of planning (Østrem, 1974).

A simple hydroelectric project will involve impounding the varying runoff in a reservoir of maximum capacity  $R$  while drawing off a constant yield of water for generation of hydroelectricity. The maximum yield is equal to the long-term mean of the runoff which flows into the reservoir. According to Hurst (1956) the storage capacity  $R$  is given by the formula:

$$R = s_q (L/2)^k \quad (6)$$

where  $L$  is the planned operating life of the reservoir and  $k$  is Hurst's parameter. If the reservoir is filled to a capacity of  $R$  before drawing off any water, a constant yield of  $\bar{q}_0$  can be maintained for the following  $L$  years without the reservoir running dry. Dividing Equation (6) by  $\bar{q}_0$  gives the relative capacity of the reservoir in relation to mean runoff:

$$\delta = R/\bar{q}_0 = \beta(L/2)^k \quad (7)$$

where  $\beta$  is the coefficient of variation of the annual runoff for a mean glacier balance of zero. As all the variables have been treated as annual quantities, the Equation (7) describes carry-over storage in the reservoir and does not include within-year storage which will depend upon seasonal variations of the inputs as well as upon any operating rules for varying the yield.

The final design for any hydropower project would be based upon runoff statistics derived from as many years of measurement as possible. However, the resulting statistics, although based upon actual measurements, would still be subject to statistical sampling errors. For example, the mean value of a past period of record would not repeat itself exactly for a future record. This effect can be expressed by a confidence interval around the computed mean runoff (Kreyszig, 1970, p. 178) given by:

$$E = \pm s_q f / \sqrt{N} \quad (8)$$

where  $N$  is the length of available record and  $f$  is the value of Student's  $t$ -statistic for  $N-1$  degrees of freedom and a preset confidence level. Dividing (8) by  $\bar{q}_0$  gives the relative error in the mean runoff with respect to the mean itself:

$$\varepsilon = E/\bar{q}_0 = \pm \beta f / \sqrt{N} \quad (9)$$

As the factor  $\varepsilon$  only decreases relatively slowly with the length of record, rather long series will be required to calculate reliable mean values for runoff.

From the above it is clear that the most important hydrological information required for preliminary design of hydropower projects are estimates of the mean runoff  $\bar{q}_0$  and the coefficient of variation  $\beta$ .

In the Appendix an attempt is made to derive a rough model for  $\beta$  whereby:

$$\beta = \gamma (\bar{p}/\bar{q}_0) \sqrt{[1 + 2\alpha^*(\alpha^*-1)]} \quad (10)$$

where  $\gamma$  is the coefficient of variation of the precipitation given by  $s_p/\bar{p}$  and  $\alpha^*$  is the effective degree of glacierization given by:

$$\alpha^* = (\bar{c}/\bar{p}) \alpha \quad (11)$$

where  $\bar{c}$  is the long-term mean of the accumulation. The accumulation (area-averaged over the whole glacier) will be generally less than the precipitation which includes both rainfall and snowfall so that  $\alpha^*$  will tend to be less than  $\alpha$ .

Equation (10) expresses the coefficient of variation of the runoff  $\beta$  as a function of the coefficient of variation of the precipitation  $\gamma$ , of the ratio of the mean precipitation to mean runoff  $\bar{p}/\bar{q}_0$  and of the effective degree of glacierization  $\alpha^*$ . The ratio  $\bar{p}/\bar{q}_0$  will tend to be largest for a glacier-free basin (because of higher evaporation) and will tend towards unity for a completely glacier-covered basin. The term under the square root will tend to a minimum for a basin with a fairly high degree of glacierization, e.g.  $\alpha$  about 0.6 to 0.8, so that glacier-cover in a basin will tend to make the runoff smoother compared to an unglacierized basin.

## TESTING OF ASSUMPTIONS AND ESTIMATION OF PARAMETERS

### Introduction

There are no long glacier runoff or mass balance series from Greenland. Therefore the preceding model cannot be tested directly under Greenland conditions. The approach adopted here is to examine the properties of mass balance series for glaciers under a wide range of conditions in the Northern Hemisphere together with precipitation series from stations on the west coast of Greenland. It is hoped that the regularities shown by these series are at least roughly valid for glaciers in Greenland as well.

Two data sets were used for the study. The first, 'World glaciological data', comprises mass balance series from 40 glaciers with at least ten years of continuous record. The glaciers come from Arctic Canada, Alaska, western North America, Scandinavia, the Alps and the USSR. For 23 out of 40 glaciers, separate series of accumulation and ablation (or winter and summer balances) are available also. The second data set, 'West Greenland precipitation data', comprises 10-year series of annual precipitation from 17 stations on the west coast of Greenland between 60 and 77°N as shown on the map in fig. 2. Details and sources of data

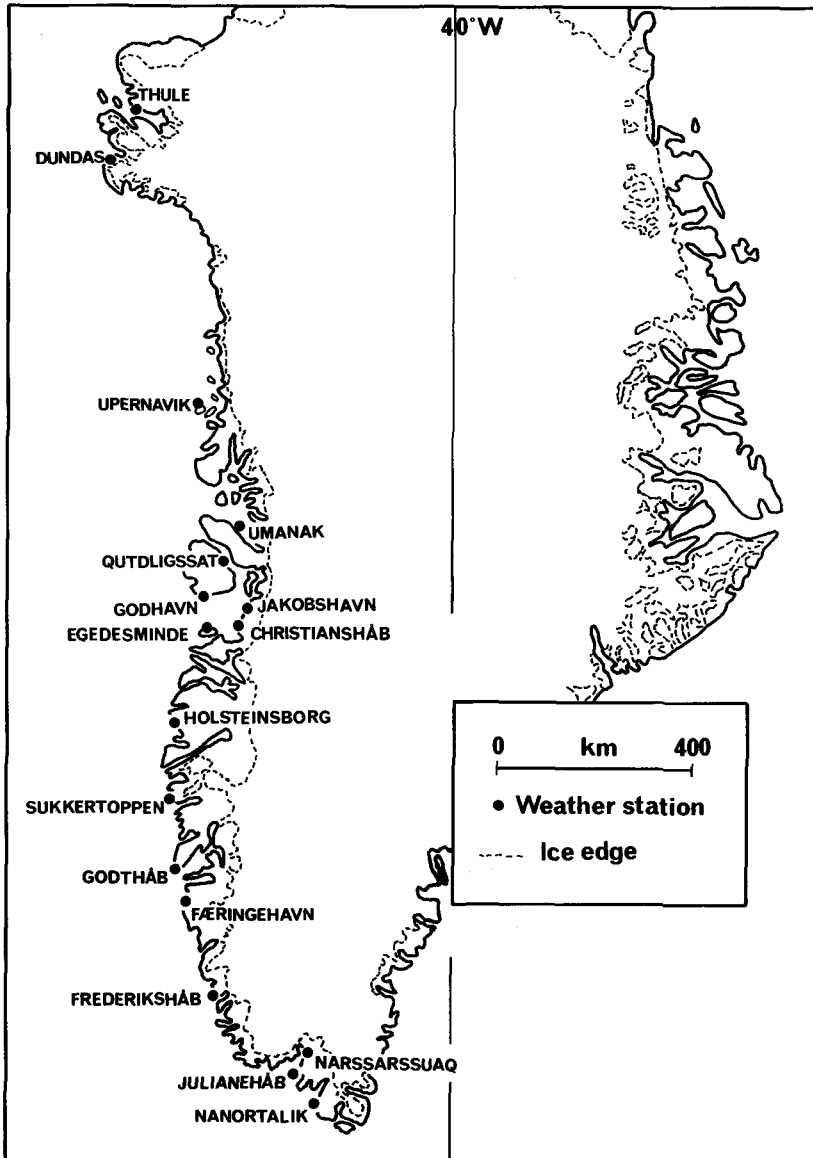


Fig. 2. Sketch map showing locations of weather stations whose data were used in the present study.

for both data sets are given in the Appendix (numbering of points in the figures follows the numbering systems in the Appendix).

The 40 glaciers cover a wide spectrum of conditions between 43 and 79°N and include both extremes of maritime and continental climate. One might, therefore, believe that the data set includes analogies of the kinds of glaciers found in West Greenland. However, the majority of the glaciers are small compared to typical lobes of the Greenland Inland Ice although the effects of size will be partly eliminated by the fact that the data refer to area-averaged values rather than to total volumes of water. In contrast, the 17 weather stations do reflect actual conditions in West Greenland although they are biased to coastal locations and may not represent average conditions for inland basins extending up to the summit of the Inland Ice.

### Mean mass balance

The mean mass balances of all 40 glaciers are plotted against the corresponding length of series in fig. 3. It is noteworthy that relatively large negative mean balances can be maintained for periods of 20 to 30 years even for small glaciers. There is no sign that balances tend to zero with increasing length of series (the apparent tendency for longer series to have more negative balances probably reflects the fact that they were started in the 1940s and 1950s which were warmer than the IHD period 1965–1974 represented by the glaciers with shorter records).

It is also noteworthy that the mean balances of most of the glaciers are not significantly different from zero at the 5 per cent level. This arises because the year-to-year fluctuations in balance are large compared to the long-term mean balances. A consequence of this is the fact that relatively long mass balance series are required before one can safely reject the hypothesis of zero balance.

### Coefficients of variation

Standard deviation are plotted against mean values for the 23 glacier accumulation and ablation series and for the 17 West Greenland precipitation series in figs. 4 to 6. The

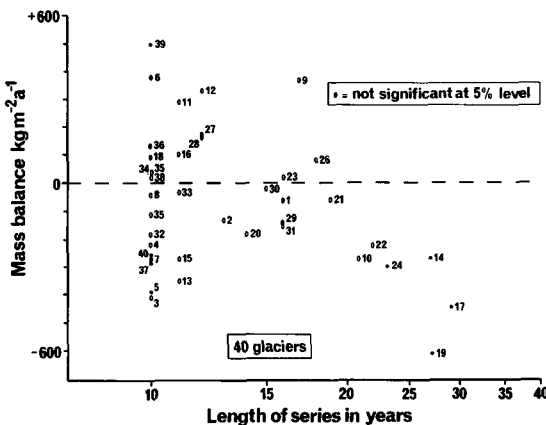


Fig. 3. Mean mass balance plotted against length of series for 40 glaciers. Open circles denote values which are not significantly different from zero at 5 per cent level.

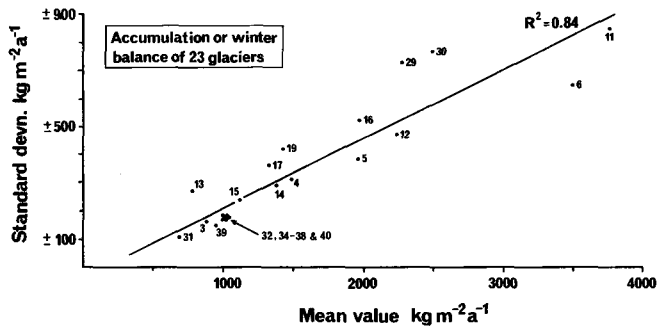


Fig. 4. Standard deviation plotted against mean value for accumulation of 23 glaciers.

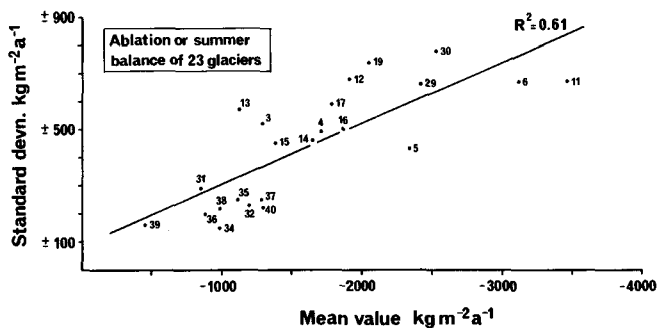


Fig. 5. Standard deviation plotted against mean value for ablation of 23 glaciers.

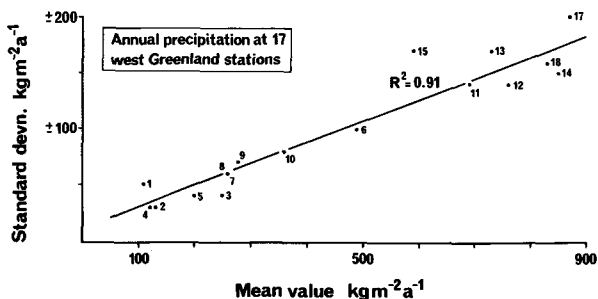


Fig. 6. Standard deviation plotted against mean value for annual precipitation at 17 West Greenland stations.

corresponding regression lines are also shown. The graphs are very similar with slopes of 0.24, 0.21 and 0.19 respectively for accumulation, ablation and precipitation whilst the intercept values are reasonably close to zero compared to the possible sampling errors. The mean values of the coefficients of variation  $\gamma$  for the individual series are given in Table 2. Once again, they lend support to the assumption in deriving Equation (10) that accumulation, ablation and precipitation have equal coefficients of variation as a first approximation.

The fact that the standard deviations of the different series bear such a definite relation to the corresponding mean values may have a statistical explanation as this is a characteristic of certain one-sided probability distributions. For example, Yevjevich (1972, p. 134–142) has suggested that the lognormal or Galton distribution is applicable to elements like precipitation and runoff which are bounded by zero.

*Table 2. Means and standard deviations of the coefficients of variation  $\gamma$ , Helmerts's parameter  $h$  and Hurst's parameter  $k$  for the different data sets*

Data set	$\gamma$	$h$	$k$
Mass balance of 40 glaciers	-	-0.12 ± 0.81	0.65 ± 0.10
Mass balance of 23 of the above glaciers	-	(-0.20 ± 0.77)	(0.63 ± 0.11)
Accumulation of the above 23 glaciers	0.22 ± 0.06	-0.13 ± 0.79	0.63 ± 0.10
Ablation of the above 23 glaciers	0.28 ± 0.09	+0.23 ± 1.14	0.70 ± 0.09
Annual precipitation at 17 West Greenland stations	0.23 ± 0.06	-0.45 ± 1.18	0.65 ± 0.10
Grand mean of the above excluding ( )	0.24 ± 0.08	+0.10 ± 0.97	0.66 ± 0.10
Sample size	63	103	103

### Standard deviation of mass balance

For the derivation of Equation (10) it was assumed that the correlation between accumulation and ablation is zero. This, together with the assumptions of zero mean balance and equal coefficients of variation for accumulation and ablation, leads to the assumption of a proportionality between the standard deviations of balance and accumulation respectively. The proportionality factor should be  $\sqrt{2}$ .

Correlation coefficients were calculated for the 23 glacier series for which separate accumulation and ablation data are available. A wide spread of values was found; a range from -0.54 to +0.62 with mean value +0.08 and standard deviation  $\pm 0.31$  for the 23 cases. To eliminate the effects of differing lengths of series, the correlations were converted to standardized values of Fisher's  $z$  (Kreyszig, 1970, p. 345) which were found to have a mean value of +0.29 with standard deviation  $\pm 1.00$ . As this mean value is not significantly different from zero at the 5 per cent level, it is difficult to reject the general hypothesis of zero correlation between accumulation and ablation. Nevertheless, many glaciologists would expect a moderately positive correlation because the ablation of a thick winter snowcover (with high albedo) should be less 'efficient' than the ablation of an equivalent mass of glacier ice (with low albedo).

The standard deviation of balance and accumulation respectively for the 23 series are compared in fig. 7 together with a line of slope  $\sqrt{2}$ . There is actually considerable scatter as well as a tendency for the points to lie above the  $\sqrt{2}$  line with a mean deviation of +15 per cent. However, a rough calculation suggests that this may mainly reflect the fact that balances are generally negative rather than zero for the sample of 23 glaciers.

### Correlation between precipitation and balance

For the derivation of Equation (10) it was assumed that the correlation between precipitation and balance was equal to  $1/\sqrt{2}$ , i.e. a value of 0.71. Braithwaite (1977, p. 106) gives

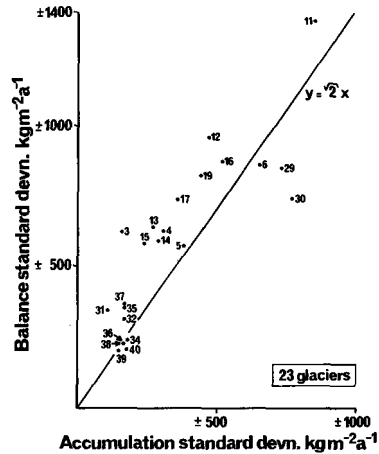


Fig. 7. Standard deviation of mass balance plotted against standard deviation of accumulation of 23 glaciers and compared with line of slope  $\sqrt{2}$ .

the calculated values found for eight Alpine glaciers. The mean correlation is +0.69 with standard deviation  $\pm 0.10$  for the eight cases which compares surprisingly well with the assumption made here.

### Homogeneity and distribution function

In the theoretical derivation of the model, elements like accumulation, ablation, balance and precipitation are assumed to be stationary random processes whilst the assumption of a normal distribution is implicit in calculating the confidence interval in Equation (9). However, as the mass balance elements are expressed as area-averages with respect to the glacier areas which change over the course of time, rigorous stationarity cannot be expected *a priori*. Randomness implies that values in any particular year are independent of values in previous years and there are plausible reasons why this may not be strictly true. Lastly, there are grounds for believing that processes like accumulation, ablation and precipitation are distributed according to one-sided functions so that the resulting balance and runoff series will not strictly follow the normal distribution.

A simple, albeit 'weak', test for homogeneity (a property combining randomness and stationarity) was applied to all the available series. This was Helmert's test (Conrad, 1944, p. 134–135) which involves counting the numbers of sequences and changes in the individual series and dividing their differences by  $\sqrt{(N-1)}$  where  $N$  is the length of series. The resulting statistic is Helmert's parameter  $h$  which will have a mean value of zero with unit standard deviation for series sampled from a homogeneous population. Values of  $h$  for the different series are summarized in Table 2. Although there is considerable scatter, the mean values of  $h$  for the different samples are never significantly different from zero at the 5 per cent level. It is therefore difficult to reject the hypothesis of homogeneity at present.

With respect to distribution function, a trial calculation of lognormal functions with coefficients of variation equal to 0.25 indicate that they are not greatly different from normal distributions except at the extrema. Rather long series, therefore, would be required to test between alternative hypotheses of normal and lognormal distributions using tests like

Chi-square. However, a Chi-square test for the normal distribution was made on the six balance series which are longer than 20 years. It was found that the hypothesis could not be rejected at the 5 per cent level in any of the cases although it should be noted that even these series (of lengths from 21 to 29 years) are still rather short for valid performance of the test given by Kreyszig (1970, p. 249). The question of the correct distribution function should, perhaps, be left open.

### Hurst's parameter

The value of Hurst's parameter  $k$  was calculated for each series by the method of Hurst (1956). This involves calculating deviations from the mean of each series and then forming a cumulative sum of the deviations. The range, or difference between maximum and minimum, of the cumulated deviations is then taken as the required storage capacity of the series which is substituted into an equation similar to (6) to obtain  $k$ . The mean values of  $k$  for the different samples are shown in Table 2 from which it can be seen that differences between samples are reasonably small. It is noteworthy that the present  $k$  values are generally smaller than the mean value of 0.73 given by Hurst (1956) as a result of analysing many different kinds of geophysical series. This is also illustrated in fig. 8 which shows the required storage (divided by the corresponding standard deviation) plotted against length of series for 40 glacier balance series together with a line for  $k = 0.73$  as given by Hurst. However, the series analysed by Hurst were longer than those discussed here, which might therefore involve greater uncertainties due to sampling, and there is furthermore the point that Hurst's equation is an empirical one so that too much attention should not be paid to the precise values of  $k$  found.

### Conclusions

The foregoing survey does not prove *sensu stricto* that the proposed model is applicable to glaciers in West Greenland. However, there seem to be certain regularities which are shared by the various series studied. One might therefore assume that glaciers in West Greenland

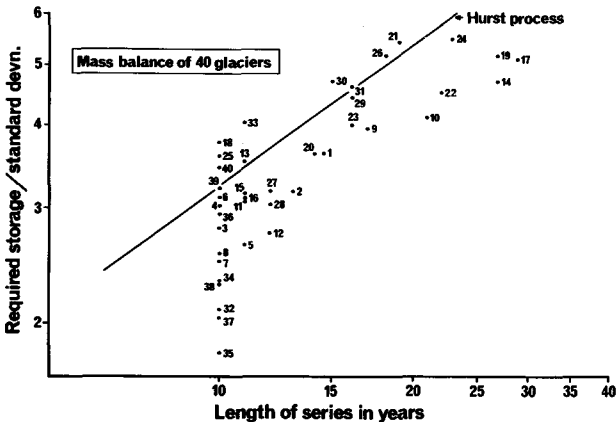


Fig. 8. Required storage divided by standard deviation plotted against length of series for mass balance of 40 glaciers and compared with the line for a Hurst process.



also share these properties until such time as enough data are available from Greenland to allow critical checking of the assumption.

## APPLICATION OF THE MODEL TO SIXTEEN PROPOSED HYDROPOWER PROJECTS IN WEST GREENLAND

### Introduction

ACG/VBB (1975) identified 16 possible sites for hydropower projects and made estimates of runoff and power production, see Table 1. The runoff values were estimated by assuming simple regional models for ablation and precipitation while evaporation was neglected. Delineation of glacier areas was only made for the assumed ablation zones. It would be of interest to apply the present model to the same sixteen basins as a comparison.

The present model approximates the runoff and its derived characteristics in terms of precipitation and evaporation so that it is necessary to specify the regional distribution of these quantities before implementing the model.

### Assumed distribution of precipitation and evaporation in West Greenland

The 1965–1974 mean annual precipitation at the 17 stations shown in fig. 2 is plotted against latitude in fig. 9. The corresponding regression equation is given by:

$$\bar{p} = 3680 - 47.8 \varphi \quad (12)$$

where  $\bar{p}$  is the mean annual precipitation in  $\text{kg m}^{-2} \text{a}^{-1}$  and  $\varphi$  is the latitude in  $^{\circ}\text{N}$ . The equation indicates a strong relation between precipitation and latitude but most of the 17 stations reflect coastal locations (see map in fig. 2) and will not describe average precipitation for inland basins extending up to the summit of the Inland Ice. There is usually a tendency for precipitation to decrease with distance from the coast, e.g. the precipitation at Narssarsuaq (point 15 in fig. 9) is markedly lower than at its close neighbours Frederikshåb, Julianehåb and Nanortalik (points, 14, 16 & 17 respectively) which are doubtlessly more

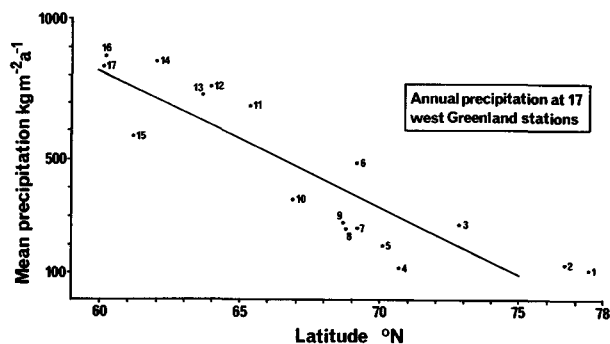


Fig. 9. Mean precipitation as a function of latitude in West Greenland.

maritime in climate. ACG/VBB (1975) also suggest that precipitation increases with elevation. However, accumulation patterns on the Inland Ice (Mock, 1967) show a tendency for accumulation to have a maximum at intermediate elevations in addition to the latitudinal trend. The effects of latitude and altitude on precipitation patterns in Greenland are certainly interesting problems to be solved for the future. However, for the present, the problem will be avoided simply by assigning generous error limits to Equation (12). An error of  $\pm 150 \text{ kg m}^{-2}$  will be assumed for the precipitation calculations (a figure which is a little larger than the root-mean-square error of Equation (12)).

The situation with regard to the regional distribution of evaporation is even less satisfactory as hardly any high-quality measurements have been made. For the purposes of the present discussion an annual evaporation of  $200 \text{ kg m}^{-2} \text{ a}^{-1}$  at latitude  $60^\circ\text{N}$  is assumed with a decrease of  $10 \text{ kg m}^{-2} \text{ a}^{-1}$  for every degree northwards (roughly the same proportional decrease as for the precipitation). These figures are pure guesses and detailed field investigations of evaporation are now being made by the Geological Survey of Greenland.

### Comparison with runoff estimates by ACG/VBB

The area and degree of glacierization for each of the 16 sites proposed by ACG/VBB (1975) were determined by planimetry of 1:250 000 maps which were drawn to depict the areas draining into the proposed reservoirs. For the preparation of these, the glacier delineations of the new glacier inventory (Weidick, personal communication) were followed although considerable adjustments had to be made for the areas above 1800 m (not included in the glacier inventory) by using areal data from the older basin inventory (Weidick & Olesen, 1978). The extent of the lower ice-free parts of the drainage areas were determined

*Table 3. Calculation of runoff for sixteen proposed hydroelectric projects using the present model*

Project	Area $\text{km}^2$	$\alpha$ %	$q_0$ $\text{kg m}^{-2} \text{ a}^{-1}$	$Q_0$ $\text{km}^3 \text{ a}^{-1}$
Motzfeldt Sø	1810 ± 380	70	700 ± 150	1.27 ± 0.38
Johan Dahl Land	350 ± 60	62	680 ± 150	0.24 ± 0.07
Grænseland	480 ± 110	74	700 ± 150	0.34 ± 0.11
Isorssua	440 ± 80	60	660 ± 150	0.29 ± 0.08
Kangarssup	3740 ± 1010	90	680 ± 150	2.54 ± 0.89
Bjørnesund	440 ± 110	83	640 ± 150	0.29 ± 0.10
Qaqat akulerit	720 ± 120	56	580 ± 150	0.42 ± 0.13
Grædefjord	3900 ± 1000	85	630 ± 150	2.45 ± 0.86
Isortuarssûp	2090 ± 450	71	590 ± 150	1.24 ± 0.41
Buksefjorden	690 ± 10	2	480 ± 150	0.33 ± 0.11
Imarsuaq	3180 ± 780	82	560 ± 150	1.78 ± 0.65
Tasersuaq	6310 ± 1160	61	520 ± 150	3.30 ± 1.13
Søndre Isortoq	4370 ± 1230	94	530 ± 150	2.33 ± 0.93
Tasersiaq	5490 ± 1170	71	470 ± 150	2.57 ± 1.00
Umivit	5930 ± 1350	76	450 ± 150	2.64 ± 1.08
Nûgssuaq	990 ± 90	32	270 ± 150	0.26 ± 0.15

Note: errors in areas are assumed to be 30 per cent of the area of ice cover.

afresh from the topography because the lower boundaries of the basins given by Weidick & Olesen (1978) are inapplicable as they relate to basins terminating at sea level. In the event, the delineations of the ice-free areas in the present study were found to be in quite close agreement with those given by ACG/VBB (1975).

After determination of the area and degree of glacierization for each basin, the corresponding runoff estimates were calculated from Equation (5) and the assumptions discussed in the previous section. The results are summarized in Table 3 and are compared with the values given by ACG/VBB (1975) in fig. 10. The error bars on the plotted points represent vectorial combinations of the estimated precipitation error ( $\pm 150 \text{ kg m}^{-2} \text{ a}^{-1}$ ) and an estimated area error of  $\pm 30$  per cent of the area of ice cover.

There is probably little basis for claiming that the results from the present model are generally better than the earlier ACG/VBB (1975) estimates. However, they do act as independent checks on the earlier values. For example, it can be seen from fig. 10 that there is generally order-of-magnitude agreement between the pairs of estimates although discrepancies can be considerable, nearly a factor of two in one case. Predictions by such simplified models as these must therefore be treated with caution. The surest way of testing the models, and hopefully improving them, is by field measurement. In fact, the Greenland Technical Organization (GTO, 1979) has started runoff and/or climate measurements in a number of the basins since the mid-1970s. However, much of the data that have appeared in unpublished technical reports appear to be preliminary, for example there appears to be a particular problem in establishing adequate rating curves to calibrate discharge records. Of the basins under investigation, Johan Dahl Land appears to be the best studied and documented. Accordingly, the runoff conditions in the area will be examined in more detail in the following section.

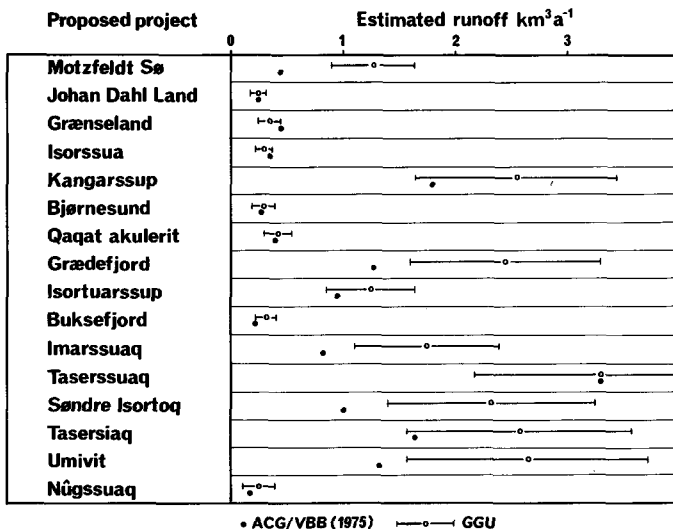


Fig. 10. Comparison between estimates of mean annual runoff by ACG/VBB (1975) and the present model for 16 proposed hydroelectric projects.

## APPLICATION OF THE MODEL TO JOHAN DAHL LAND

## Introduction

Johan Dahl Land, a few kilometres north of the Narssarssuaq weather station in South Greenland at 61.2 °N (see map, fig. 2), is one of the basins where a hydropower project has been proposed (ACG/VBB, 1975). The runoff from the Nordbosø (a natural lake which would be regulated as a reservoir) has been measured since 1976 while measurements of runoff from the Thor Sø (which would be connected to the Nordbosø by a tunnel) were started in 1978. The annual runoff from the two basins in  $\text{km}^3 \text{a}^{-1}$  are shown in Table 4. The figures are the latest, unpublished, values from the GTO (calculated in autumn 1981) which supersede earlier versions that have appeared in unpublished reports. Apparently it has been quite problematic to establish a rating curve for the streams.

Studies on the Nordbogletscher (the largest glacier draining into the Nordbosø) were started in 1977 (Olesen & Weidick, 1978) and several years of mass balance data are now available. Clement (1982) estimated that the Nordbogletscher was close to balance for 1978/79 whilst the measured balances for 1979/80 and 1980/81 were  $-290$  and  $-250 \text{ kg m}^{-2} \text{a}^{-1}$  for an area of  $208 \text{ km}^2$ .

On the basis of the few years of available runoff data (unfortunately, the earlier uncorrected values) ACG/VBB (1980) prepared an outline of the planned Johan Dahl Land project. As a comparison, it would be interesting to apply the present model, i.e. Equations (5) to (10), to the area. The results obtained are given in Table 5. The mean precipitation  $\bar{p}$ , coefficient of variation  $\gamma$  and Hurst's parameter  $k$  are those calculated for the 1965–1974 precipitation record at Narssarssuaq whilst the evaporation  $\bar{e}$  is a guess. The areas of the sub-basins for Nordbosø and Thor Sø are taken from Olesen & Weidick (1978). The calculated quantities are the specific runoff  $\bar{q}_0$ , the total runoff  $\bar{Q}_0$ , coefficient of variation  $\gamma$ , relative reservoir capacity for 30-year storage  $\delta$ , and relative error in the 10-year mean of the measured runoff  $\epsilon$ .

For the calculations in Table 5 the choice of 0.83 for the ratio  $\bar{e}/\bar{p}$  requires further discussion. It is estimated by assuming that the accumulation in the accumulation zone is

Table 4. Total runoff from the Nordbosø and Thor Sø sub-basins

Units are  $\text{km}^3 \text{a}^{-1}$

Year	Nordbosø 308 km <sup>2</sup>	Thor Sø 33 km <sup>2</sup>	Combined 341 km <sup>2</sup>
1976	0.155	-	-
1977	0.133	-	-
1978	0.142	0.020	0.162
1979	0.125	0.012	0.137
1980	0.132	0.012	0.144
Mean	0.137	0.015	0.148
s.d.	0.012	0.005	0.013

Based on GTO unpublished data.

Table 5. Calculation of runoff and its characteristics for Johan Dahl Land, southern West Greenland

	Nordbosø	Thor sø	Combined
$F_B$ (km <sup>2</sup> )	308	33	341
$\alpha$	0.69	0.00	0.62
$\bar{c}/\bar{p}$	0.83	0.00	0.83
$\alpha^*$	0.56	0.00	0.51
$\bar{q}_o$ (kg m <sup>-2</sup> a <sup>-1</sup> )	530	390	520
$\bar{Q}_o$ (km <sup>3</sup> a <sup>-1</sup> )	0.163	0.013	0.176
$\beta$	0.23	0.44	0.24
$\delta$ (30-year)	1.30	2.49	1.36
$\epsilon$ (10-year)	$\pm 0.13$	$\pm 0.26$	$\pm 0.14$

$\bar{p} = 590$  kg m<sup>-2</sup> a<sup>-1</sup> (Narssarssuaq data 1965–74)

$\gamma = 0.29$  (Narssarssuaq data 1965–74)

$k = 0.64$  (Narssarssuaq data 1965–74)

$\bar{e} = 200$  kg m<sup>-2</sup> a<sup>-1</sup> (guessed value)

Note: areal data  $F_B$  and  $\alpha$  are based upon Olesen & Weidick (1978).

identical to the precipitation while the accumulation in the ablation zone is one half of the precipitation to take account of the fact that precipitation falls as rain as well as snow (rainfall in the accumulation zone is assumed to be retained by refreezing). If the area of the accumulation zone is assumed to be twice that of the ablation zone (Braithwaite & Müller, 1980) a value of 2.5/3 or 0.83 is obtained for the ratio  $\bar{c}/\bar{p}$ . This may seem very arbitrary. However, a trial calculation shows that  $\beta$  is not very sensitive to the exact choice of ratio as long as it is reasonably large, e.g. greater than 0.5, and as long as the degree of glacierization is in the range 0.5 to 0.8. This is illustrated by the figures in Table 6 which are based upon the same precipitation and evaporation values as Table 5.

Table 6. Variation of  $\beta$  as a function of the ratio  $\bar{c}/\bar{p}$  and degree of glacierization  $\alpha$

$\bar{c}/\bar{p}$	$\alpha = 0.0$	$\alpha = 0.33$	$\alpha = 0.67$	$\alpha = 1.0$
0.0	0.44	0.38	0.33	0.29
0.2	0.44	0.36	0.29	0.24
0.4	0.44	0.33	0.26	0.21
0.6	0.44	0.31	0.24	0.21
0.8	0.44	0.30	0.23	0.24
1.0	0.44	0.28	0.25	0.29

### Mean runoff

According to the results of the model shown in Table 5, the Nordbosø and Thor Sø basins together should have a combined runoff of  $0.176 \text{ km}^3 \text{ a}^{-1}$  with coefficient of variation 0.24. By comparison, the corresponding mean measured runoff for 1978–1980 is  $0.148 \text{ km}^3 \text{ a}^{-1}$  (Table 4). The model has overestimated by 19 per cent. However, if one considers that the model purports to estimate runoff for a glacier balance of zero, it might be more meaningful to compare the model with the measured runoff for 1979 when the glacier is estimated to be close to zero balance. In this case the model overestimates the 1979 runoff by 41 per cent.

The above errors seem quite large. However, ACG/VBB (1975) estimated the runoff to be  $0.25 \text{ km}^3 \text{ a}^{-1}$  whilst the version of the present model which uses the regional precipitation distribution gives a value of  $0.24 \text{ km}^3 \text{ a}^{-1}$ . These are overestimates of 69 and 62 per cent respectively compared to the measured mean of  $0.148 \text{ km}^3 \text{ a}^{-1}$ . The comparison of these figures with those in Table 5 shows, among other things, the importance of using Narssarsuaq precipitation data, i.e. data from a local station, rather than the generalized regional precipitation.

Considering the two basins separately, the model in Table 5 predicts runoff values of  $0.163$  and  $0.013 \text{ km}^3 \text{ a}^{-1}$  for Nordbosø and Thor Sø respectively. The observed runoffs are  $0.137$  (1976–1980) and  $0.015$  (1978–1980), i.e. the model overestimates by 19 per cent for Nordbosø and underestimates by 13 per cent for Thor Sø. The latter underestimation might be due to a lower evaporation than assumed or to a possible increase of precipitation with elevation above Narssarsuaq or a combination of the two effects. The overestimation of the model for Nordbosø is more problematic as the above factors, combined with the fact that the glacier had a generally negative balance during the period, would all lead to an even greater difference between the model and the measurements. However, Clement (1982) has also found that the measured summer balance on the Nordbogletscher alone actually exceeds the measured runoff from Nordbosø which should be fed by not only the Nordbogletscher but also by about  $100 \text{ km}^2$  of glacier-free land. A possible explanation is that part of the Nordbogletscher, as presently delineated, is not draining into the Nordbosø, i.e. the sub-glacial drainage pattern does not necessarily coincide with the surface pattern (Röthlisberger, personal communication).

More detailed field investigations on the Nordbogletscher will be required to resolve the question of the correct drainage area. From the point of view of the present model, the example is a useful warning that similar problems may arise in other areas. However, considering all the possible errors and the fact that the model is based upon generalizations and climatic data from a weather station at some distance from Johan Dahl Land, the model may be considered to give acceptable first predictions for the Johan Dahl Land runoff. It should be tested in other areas when accurate runoff data become available.

### Coefficient of variation of runoff

According to the results in Table 5, the runoff from the Nordbosø should have a lower variability than that from the Thor Sø, i.e. coefficients of variation of 0.23 compared to 0.44. The measured series are still too short to test this prediction strictly. However, the 5-year Nordbosø series in Table 4 has a coefficient of variation of about 9 per cent while the corresponding value for the 3-year Thor Sø series is 33 per cent. These figures at least

indicate that the model predictions are in the right direction, i.e. that the runoff from a glacierized basin is less variable than that from a glacier-free basin.

### Relative reservoir capacity

According to Table 5 the relative reservoir capacity  $\delta$  is 1.36 to maintain a steady yield equal to  $\bar{q}_0$  from the combined basins over a 30-year period. Taking the runoff to be  $0.148 \text{ km}^3 \text{ a}^{-1}$  gives a reservoir volume of  $0.20 \text{ km}^3$ . The area of the Nordbosø is about  $10.5 \text{ km}^2$  so that this storage requirement can be met by a total water level variation of about 19 m between high and low water. This is quite close to the range +3.5 to -19 m which is envisaged by ACG/VBB (1980). Their proposed reservoir should therefore be adequate for greater than 30-year storage according to the present model. However, this conclusion might have to be modified to take account of seasonal storage which is neglected in the present model.

### Relative error in mean runoff and design yield

The final design of the Johan Dahl Land project should be based, as much as possible, upon statistics of measured runoff rather than modelled runoff. However, according to Table 5, even if ten years of measurements were to be available at the time of preparing a final design (for calculations to be made in 1985), the 10-year mean runoff would still have a relative error  $\epsilon$  of about  $\pm 14$  per cent at the 90 per cent probability level. Alternatively, if the design yield of the project is set at 86 per cent of the mean, there would be about 5 per cent risk that this figure would still be too high.

A record length of ten years was assumed for the calculation of  $\epsilon$  as this was thought to be a realistic figure from the planning point of view. In fact, the magnitude of  $\epsilon$  only diminishes slowly with increasing length of record. This is illustrated in Table 7 which shows the safe yield (5 per cent risk level) as a function of record length. It can be readily seen that the extra benefit obtained, i.e. increase in safe runoff diminishes for each additional year of record. On the other hand, the direct cost of each additional year of measurement will be roughly constant. It might be best, therefore, to base the final design upon a fairly short record and to accept a correspondingly low design yield, for example ten years of record and a design yield

Table 7. Calculated 'safe' runoff from Johan Dahl Land at 5 per cent risk level as a function of length of record

Length of record Years	'Safe' runoff % of mean	Increase % of mean
5	77	-
10	86	+9
15	89	+3
20	91	+2
25	92	+1
30	93	+1

of 86 per cent with a 5 per cent risk. ACG/VBB (1980) envisage a yield of 95 per cent with a 36 per cent risk.

For a mean runoff of about  $0.15 \text{ km}^3 \text{ a}^{-1}$ , an 86 per cent design yield would be about  $0.13 \text{ km}^3 \text{ a}^{-1}$  which would give an annual power production of about  $190 \text{ GWh a}^{-1}$ . This can be compared to previous estimates of 370 (ACG/VBB, 1975) and  $220 \text{ GWh a}^{-1}$  (ACG/VBB, 1980) respectively.

The production of the Johan Dahl Land project might be even lower than the above figures if environmental factors have to be considered. For example, if it is necessary to maintain a minimum water flow in streams to preserve biological quality the required water would have to be taken from the safe runoff, i.e. less water would be available then for power production.

## FUTURE OUTLOOK

The present model is still rather crude. The main shortcomings probably reflect the present lack of knowledge of regional variations of precipitation and evaporation in Greenland. However, comparison between the model and the limited amount of data available from Johan Dahl Land suggest it can still give useful results. The model could therefore be adopted to replace the older ACG/VBB (1975) model until it is replaced in its turn by something better.

It is clear that an improved knowledge of the spatial distributions of precipitation, glacier balance, and evaporation is needed on both the regional (basin-to-basin) and local (within-basin) scales. This can only be achieved by a carefully balanced field programme which will include long-term monitoring of runoff, climate, and glacier balance as well as detailed research into specific aspects of hydrometeorology, e.g. evaporation and energy balance. Naturally, such investigations could never be made in all 870 basins of West Greenland with a total area of  $484\,000 \text{ km}^2$ . However, if only a few basins can be studied, it should be possible to make transfers of information from them to other, unstudied, basins by suitable scaling. For this purpose, the three glaciers presently studied by the Geological Survey of Greenland will give a useful, although minimum, set of point-values while the areal data contained in the basin/glacier inventories will be useful for transferring this information to other basins.



## APPENDIX

### The coefficient of variation of annual runoff

For the calculation, temporal variations in evaporation are neglected compared to variations in the other elements. From Equation (1) the variance  $s_q^2$  of the annual runoff series will then be given by:

$$s_q^2 = s_p^2 + \alpha^2 s_b^2 - 2\alpha s_p s_b r_{pb} \quad (13)$$

where  $r_{pb}$  is the correlation between the balance and the precipitation. By definition the balance in the  $t$ th year is given by:

$$b_t = c_t + a_t \quad (14)$$

where  $c_t$  and  $a_t$  are the specific accumulation and ablation respectively (area-averaged over the whole glacier). The variance of the balance  $s_b^2$  is given by:

$$s_b^2 = s_c^2 + s_a^2 + 2s_c s_a r_{ca} \quad (15)$$

where  $r_{ca}$  is the correlation between accumulation and ablation. It can be shown that the correlation between balance and precipitation  $r_{pb}$  is given by:

$$r_{pb} = (s_c/s_b)r_{pc} + (s_a/s_b)r_{pa} \quad (16)$$

where  $r_{pc}$  and  $r_{pa}$  are correlations of precipitation with accumulation and ablation respectively.

An important assumption is introduced that the coefficients of variation of accumulation, ablation and precipitation are all identical and given by:

$$\gamma = s_c/\bar{c} = s_a/\bar{a} = s_p/\bar{p} \quad (17)$$

Assuming that the mean balance is zero so that  $\bar{c} = \bar{a}$ , the Equation (17) implies that  $s_a = s_c$ . With the further assumption that  $r_{ca}$  is zero, the Equation (15) gives:

$$s_b^2 = 2s_c^2 \quad (18)$$

Substitution of (18) into (16) gives:

$$r_{pb} = (1/\sqrt{2})(r_{pc} + r_{pa}) \quad (19)$$

There should be a high positive correlation  $r_{pc}$  between precipitation and accumulation whilst a weak positive (recall that ablation is negative by definition) correlation  $r_{pa}$  between precipitation and ablation might also be expected. As a rough approximation the sum of the two correlations will be taken as unity so that:

$$r_{pb} = 1/\sqrt{2} \quad (20)$$

Substitution of Equations (18) and (20) into (13) gives:

$$s_q^2 = s_p^2 + 2\alpha^2 s_c^2 - 2\alpha s_p s_c \quad (21)$$

By replacing the standard deviations with mean values according to Equation (17) the following expression is obtained:

$$s_q = \gamma \bar{p} \sqrt{[1 + 2\alpha^*(\alpha^* - 1)]} \quad (22)$$

where  $\alpha^*$  is given by:

$$\alpha^* = (\bar{c}/\bar{p})\alpha \quad (23)$$

The coefficient of variation  $\beta$  is then simply obtained by dividing Equation (22) by the mean runoff  $\bar{q}_0$ :

$$\beta = \gamma (\bar{p}/\bar{q}_0) \sqrt{[1 + 2\alpha^*(\alpha^* - 1)]} \quad (24)$$

### Notes on world glaciological data used in the study

The main sources of data for the 40 mass balance series are the publications of the Permanent Service on the Fluctuations of Glaciers (PSFG); Kasser (1967, 1973) and Müller (1977). Some information about the series used are given in Table 8 including the period of record analysed for the present report, glacier area, latitude and responsible agency, together with an indication of whether separate accumulation (c) and ablation (a) data are available. Fuller details are given in Müller (1977). It should be mentioned that the three publications cited contain numerous small errors in addition to measurement errors inherent in glaciological quantities. Further, it is common for published glaciological data to be 'amended' later so that several versions of the same data may exist.

The periods of record in Table 8 refer to continuous records actually analysed in the present study.

Table 8. Glacier data used in the present study

No. Glacier	Period	Area (km <sup>2</sup> )	Lat. (°N)	c	a	b	Agency
<i>Canada</i>							
01 White Glacier	1960-75	38.9	79			x	McGill Univ. & ETH, Zürich
02 Baby Glacier	1960-72	0.6	79			x	
03 Ram River	1966-75	1.8	52	x	x	x	Glaciology Division, Ottawa
04 Peyto Glacier	1966-75	13.4	52	x	x	x	
05 Place Glacier	1965-75	4.0	50	x	x	x	
06 Sentinel Gl.	1966-75	2.0	50	x	x	x	
<i>USA</i>							
07 Gulkana	1966-75	19.3	63			x	US Geological Survey (USGS)
08 Wolverine	1966-75	17.7	60			x	
09 Blue Glacier	1958-74	4.2	48			x	Washington Univ. USGS
10 South Cascade	1955-75	2.7	48			x	
<i>Norway</i>							
11 Älfotbreen	1965-75	4.8	62	x	x	x	Electricity & Water Board (NVE)
12 Nigardsbreen	1964-75	46.6	62	x	x	x	
13 Grasubreen	1965-75	2.5	62	x	x	x	Polarinstitutt NVE
14 Storbreen	1949-75	5.4	62	x	x	x	
15 Hellstugubr.	1965-75	3.3	62	x	x	x	
16 Hardangerjøk.	1965-75	17.4	61	x	x	x	
<i>Sweden</i>							
17 Storglaciären	1947-75	2.1	68	x	x	x	Stockholm Univ.
<i>France</i>							
18 Saint-Sorlin	1966-75	3.5	45			x	Grenoble Univ. Min. Agriculture
19 Sarennes	1949-75	0.8	45	x	x	x	
<i>Switzerland</i>							
20 Griesgletscher	1962-75	6.3	46			x	ETH Zürich
21 Gr. Aletsch	1957-75	128.5	46			x	
22 Limmerngl.	1954-75	3.3	47			x	
23 Silvrettagl.	1960-75	3.3	47			x	
<i>Austria</i>							
24 Hintereisf.	1953-75	9.0	47			x	Innsbruck Univ. Acad. Science München
25 Vernagtferner	1966-75	9.6	47			x	
26 Kesselwandf.	1958-75	4.2	47			x	Innsbruck Univ. Salzburg Univ. Salzburg Univ.
27 Sonnblick Kess	1964-75	1.8	47			x	
28 Filleck Kess	1964-75	0.1	47			x	
<i>USSR</i>							
29 Lednik IGAN	1959-74	0.8	68	x	x	x	Acad. Science, USSR
30 Obrucheva	1960-74	0.3	68	x	x	x	
31 Karabatkak	1959-74	4.1	42	x	x	x	Acad. Kirghiz SSR
32 Igly Tuyuksu	1965-74	1.7	43	x	x	x	
33 Tsent. Tuyuksu	1957-67	3.1	43			x	Acad. Science, Kazakhian, SSR
34 Kosmodemyansk.	1965-74	0.4		x	x	x	
35 Mametovoy	1965-74	0.4	43	x	x	x	
36 Mayakovsk.	1965-74	0.2		x	x	x	
37 Molodezhnyy	1965-74	1.4	43	x	x	x	
38 Orozhonikidze	1965-74	0.3		x	x	x	
39 Partizan	1965-74	0.1		x	x	x	
40 Visychiye	1965-74	0.3		x	x	x	

Availability of accumulation c, ablation a and balance b, denoted by x.

Records for some of the glaciers are longer than shown but include breaks or serious ambiguities so that parts of the records were excluded. For the study no distinction has been made between measurements in the stratigraphic and fixed-date systems respectively so that in some cases, for example, 'ablation' refers to annual ablation and in other cases to summer balance. The distinction is probably not important in the present context and, generally, appears to be more ideal than real as a number of contributors to Müller (1977) declined to specify which system was used. All the data refer to values area-averaged over the whole glacier and are expressed in  $\text{kg m}^{-2} \text{a}^{-1}$ .

### Notes on West Greenland precipitation data used in the study

The names and locations of the 17 stations are given in Table 9. The data refer to annual precipitation totals in  $\text{kg m}^{-2} \text{a}^{-1}$  and are extracted from 'Provisional mean temperatures and total amount of precipitation in mm, Greenland' which is published periodically by the Danish Meteorological Institute in Copenhagen. The base period chosen for the data set is the 10-year IHD period 1965–1974 but some of the records are shorter due to station mutations.

The majority of the stations lie on the west coast itself and probably reflect more maritime conditions than prevail in inland basins extending to the summit of the Inland Ice. However, it would be misleading to characterize the stations by the classical continentality index (Conrad, 1944) which is based upon the annual temperature range at each station because winter temperatures at the more northerly stations are depressed by the effects of sea ice cover. Examination of the plot in Fig. 9 does prompt the idea that the point distribution would be very well described by two, nearly parallel, regression lines instead of the one used and that the lower points (including Narssarsuaq) may belong to a more 'inland' sample than the higher points. The physical basis of any such stratification is lacking at present and it is not attempted. However, the matter will be investigated further to attempt to find a rational index of 'coast/inland' effect.

*Table 9. West Greenland precipitation data used in the present study*

No. Station	Period	Lat. °N	Long. °W	$\bar{p}$	$s_p$
01 Thule-Qânâq	1966-74	77.5	69.2	(110)	(50)
02 Dundas	1965-74	76.6	68.8	130	30
03 Upernavik	1965-74	72.8	56.2	250	40
04 Umanak	1965-74	70.7	52.0	120	30
05 Qutdligssat	1965-71	70.1	52.9	(200)	(40)
06 Godhavn	1965-74	69.2	53.5	490	100
07 Jakobshavn	1965-74	69.2	51.1	260	60
08 Christianshåb	1965-74	68.8	51.1	260	60
09 Egedesminde	1965-74	68.7	52.8	280	70
10 Holsteinsborg	1965-74	66.9	53.7	360	80
11 Sukkertoppen	1965-74	65.4	52.9	690	140
12 Godthåb	1965-74	64.0	51.8	760	140
13 Færingehavn	1965-74	63.7	51.6	(730)	(170)
14 Frederikshåb	1965-74	62.0	49.7	850	150
15 Narssarsuaq	1965-74	61.2	45.4	590	170
16 Julianehåb	1965-74	60.2	46.1	870	200
17 Nanortalik	1965-74	60.1	45.2	830	160

( ) = statistics based upon less than 10-year record.

$\bar{p}$  is mean and  $s_p$  is standard deviation of annual precipitation in  $\text{kg m}^{-2} \text{a}^{-1}$ .

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